Sample entropy for graph signals: An approach to nonlinear analysis of graph signals

Authors: Maggie Lei, John Fabila-Carrasco, Javier Escudero

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Presentation by: Maggie Lei

Email: M.S.Lei@sms.ed.ac.uk

THE UNIVERSITY of EDINBURGH
Institute for Imaging, Data and Communications

AGENDA

- Introduction
- Theoretical framework
- Validation in 1D, 2D, graph signals
- Real-world traffic dataset
- Conclusion

INTRODUCTION

Real-world complex systems exhibit nonlinear dynamics.

- Brain networks
- Power grids
- Traffic network
- Social interactions

Complexity & state transition characterisationin graph signals

SAMPLE ENTROPY

Since 2000 ⇒ Canonical in time series

Applications:


Cardiology


Neuroscience


Engineering


Finance

> 6,100 citations

GRAPH-ENTROPY MEASURE

Dispersion Entropy (DE)

measures distribution pattern of symbols (Shannon entropy).

Recently
⇒ Graph-based DE (DEG)

Sample Entropy (SampEn)

conditional entropy - based metric measuring **irregularity** and **repetition** of signal patterns.

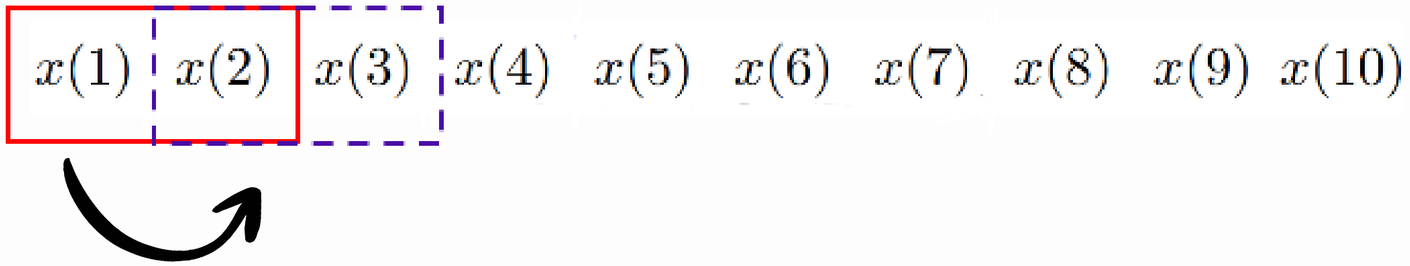
Can we define an analogous Sample Entropy measure for graph signals?

METHODS

CLASSICAL SAMPLE ENTROPY

1 Create patterns of length m

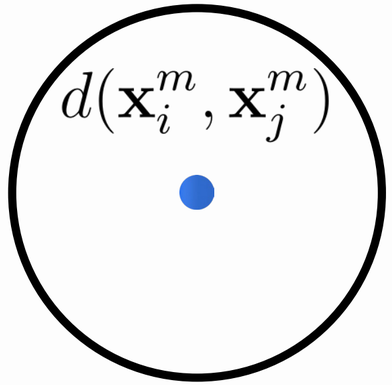
m-length pattern



2 Compute pairwise Chebyshev distance between patterns

Count similarity patterns (match)

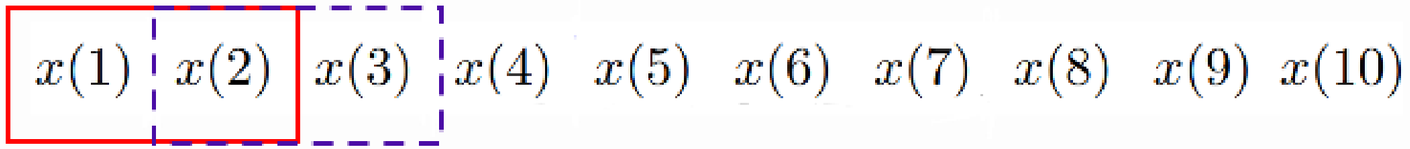
$$\text{if} \leqslant \text{threshold}$$



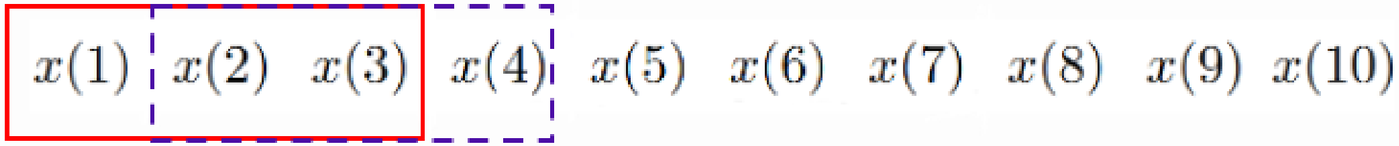
Threshold ϵ

CLASSICAL SAMPLE ENTROPY

m-length pattern



(m+1)-length pattern



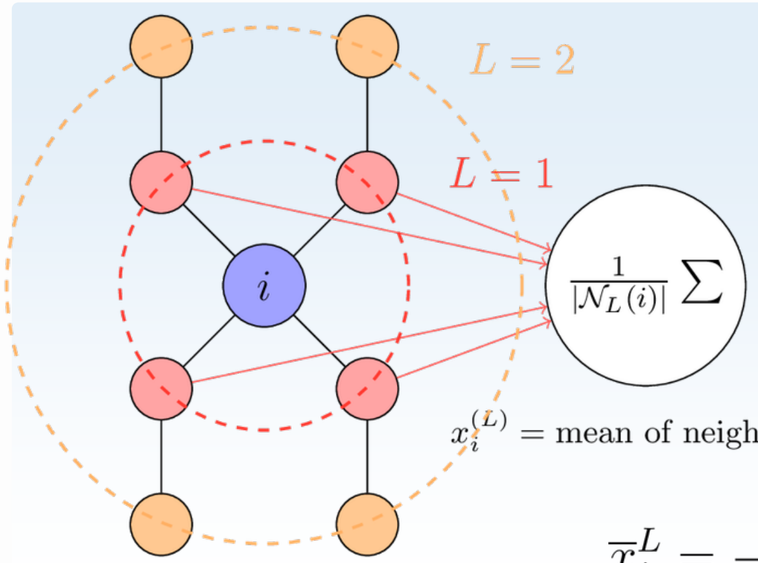
$$-\ln\left(\frac{A^m}{B^m}\right)$$

How many of these matches retain?

GRAPH-SIGNAL FRAMEWORK

L-hop Neighbourhood aggregation

$$\overline{\mathbf{x}}^m(i) = [\overline{x}_i^0, \overline{x}_i^1, \ldots, \overline{x}_i^{m-1}]$$



Graph embedding (pattern) for node i

$[\ \bar{x}_i^0,\ \bar{x}_i^1,\ \bar{x}_i^2\ ]$

$$\overline{x}_i^L = \frac{1}{\deg^L(i)} \sum_{j \in \mathcal{N}_L(i)} (\mathbf{A}^L)_{ij} x_j.$$

Measure similarity (against threshold)

$$d(\mathbf{x}_i^m, \mathbf{x}_j^m)$$

match ✓ if $d \leq \epsilon$

$$\epsilon = r \cdot \mathrm{SD}$$

Compute entropy change

when $m \to m+1$

$$-\ln\left(\frac{A^m}{B^m}\right)$$

B^m(matches at m)

A^m(matches at $m+1$)

Generalizes Sample Entropy. Continuous entropy metric on graph signals.

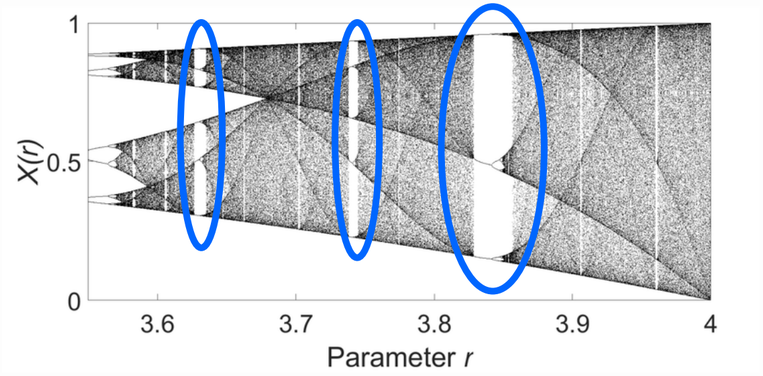
EXPERIMENTS & RESULTS

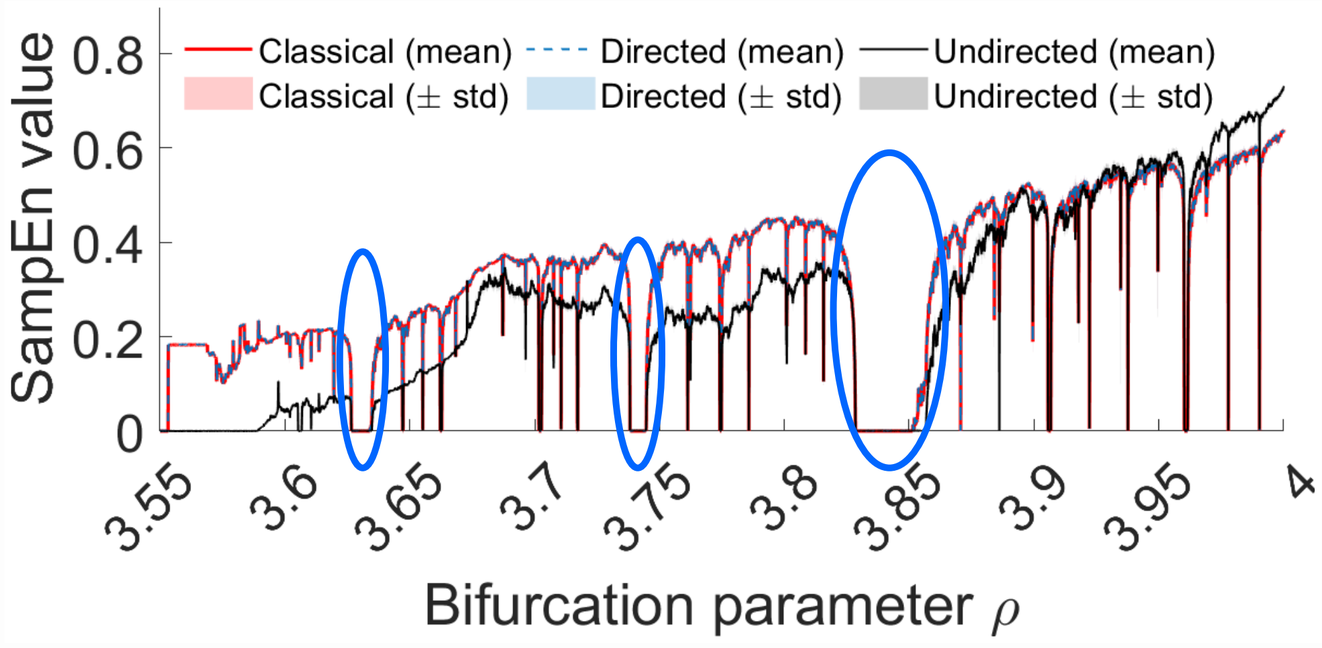
VALIDATION

Generalisability to 1D and 2D

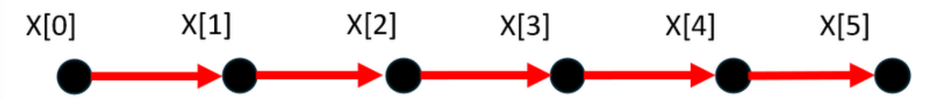
1D Logistic map $x_{n+1} = r x_n (1 - x_n)$

Islands of stability





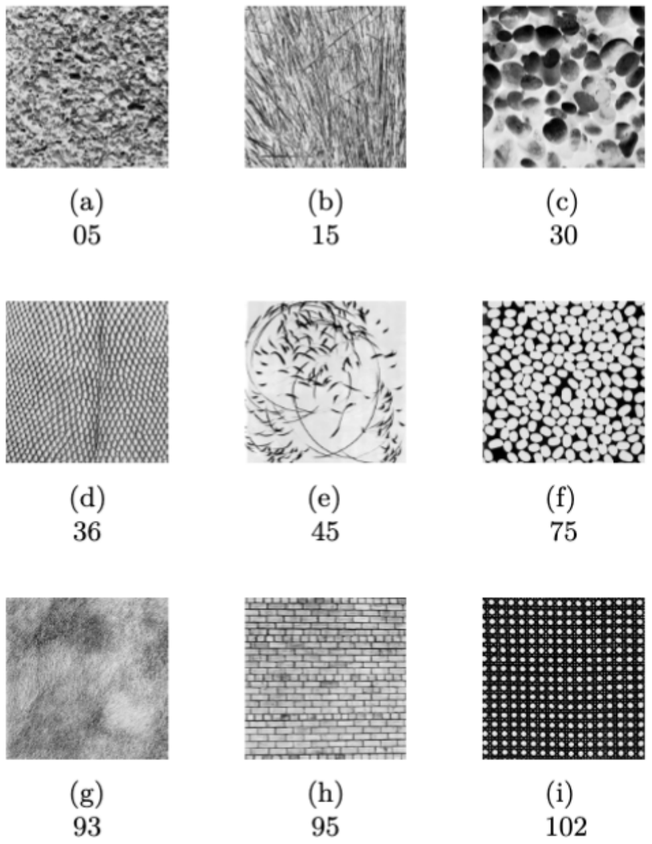
Directed path



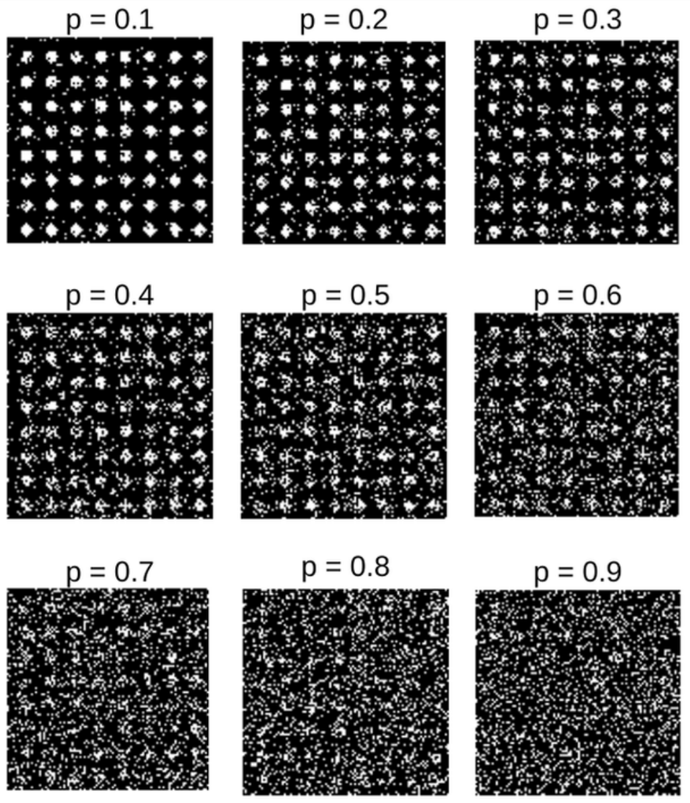
VALIDATION

Generalisability to 1D and 2D

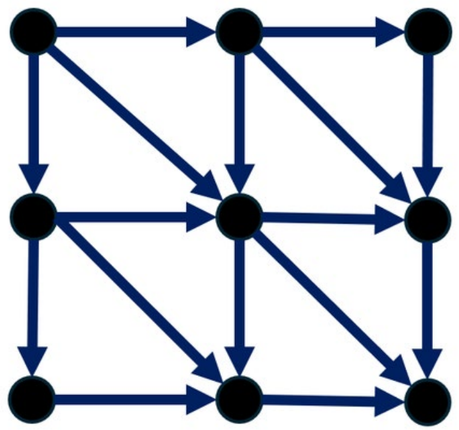
2D Brodatz dataset



MIX 2D



2D directed grids

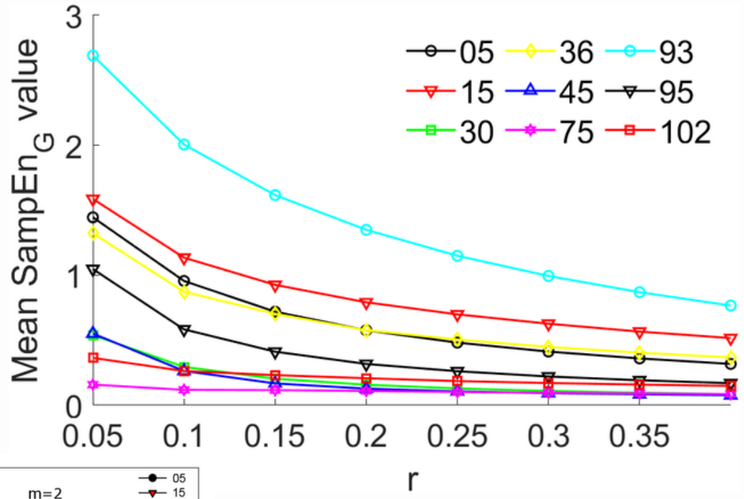


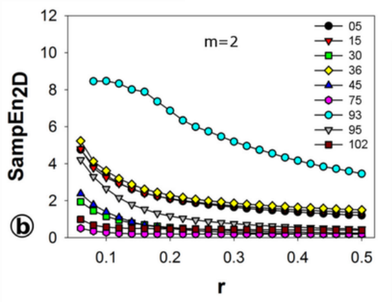
(i, j)

VALIDATION

Generalisability to 1D and 2D

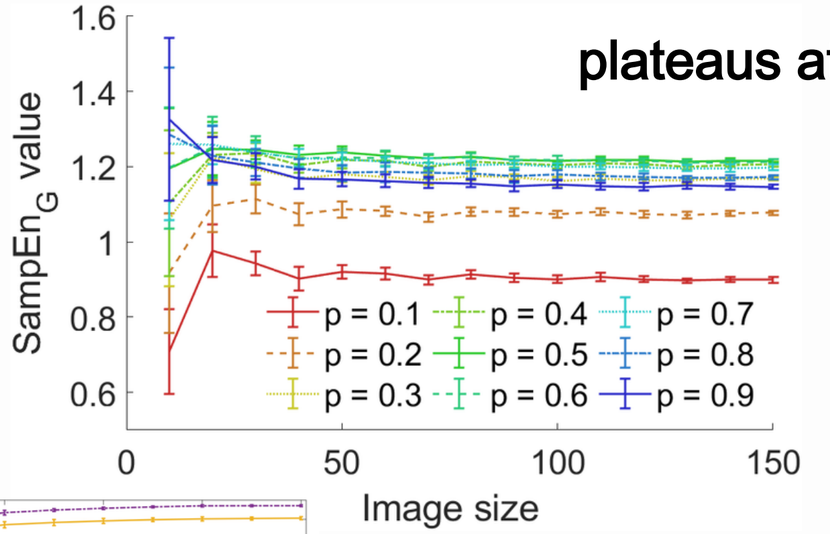
2D Brodatz dataset



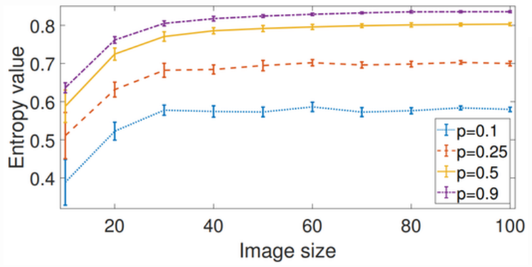


SampEn2D[1]

MIX 2D



plateaus at p>=0.5



Graph-based permutation entropy on MIX2D[2]

Follows overall SampEn2D behaviour on grids.

[1] L E V Silva *et al* 2016 *Biomed. Phys. Eng. Express* **2** 045002

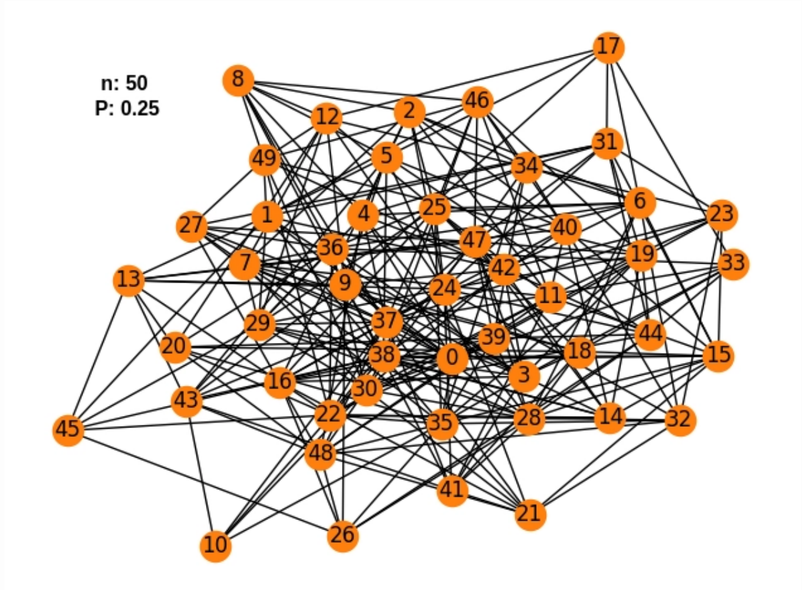
[2] Fabila Carrasco, JS, Tan, C & Escudero, J 2022, 'Permutation Entropy for Graph Signals', IEEE Transactions on Signal and Information Processing Over Networks, vol. 8, pp. 288-300

ERDŐS-RÉNYI MODEL

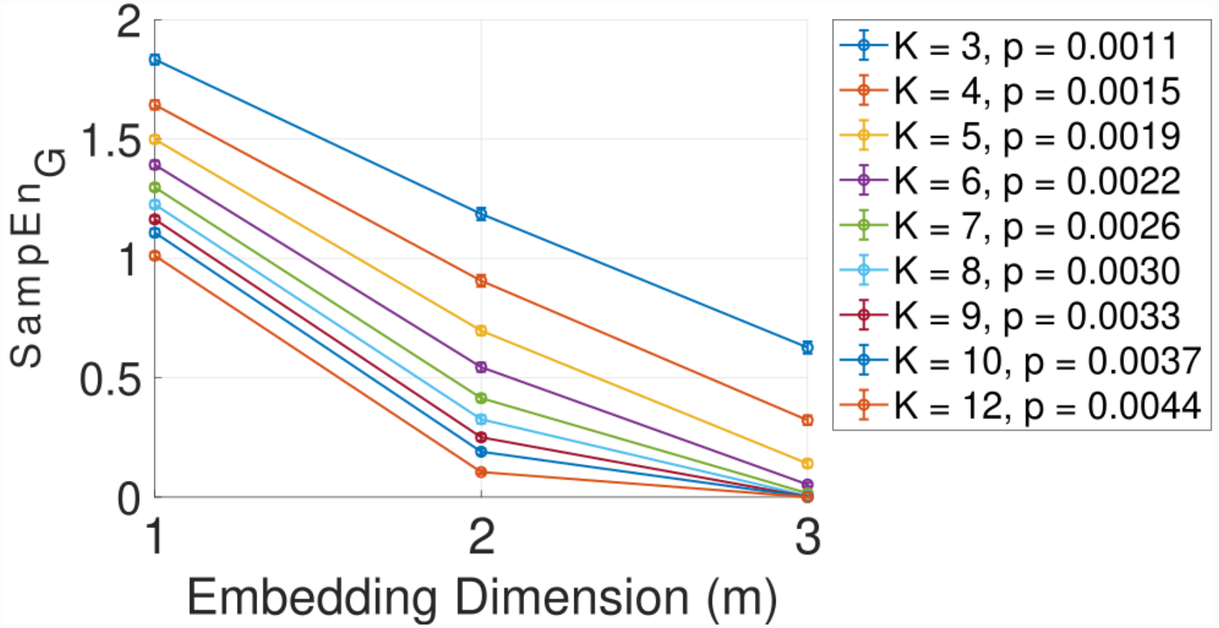
$$K \in \{3, 4, 5, 6, 7, 8, 9, 10, 12\}$$

Connectivity

$$p = \frac{K}{N-1},$$



Random signal $\in \{0.01, 0.1\}$



N=2700

REAL-WORLD DATASET

FT-AED traffic sensor dataset [3]

196 traffic sensors, 30 seconds interval over 20 weekdays

~3.76 million speed measurements

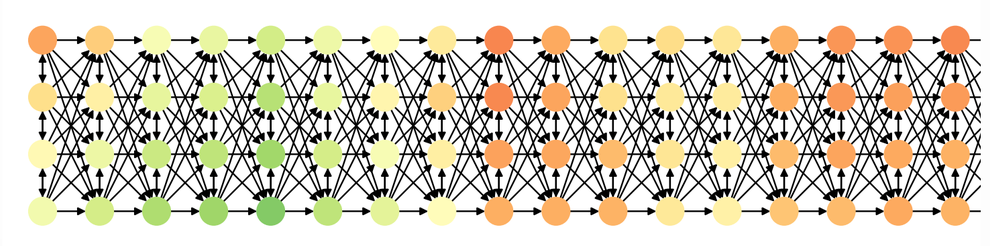
18-mile section, 4-lane freeway

Following graph construction from [3]

Nodes: traffic sensors

Edges: spatial distance

Graph signal: speeds on nodes at time t



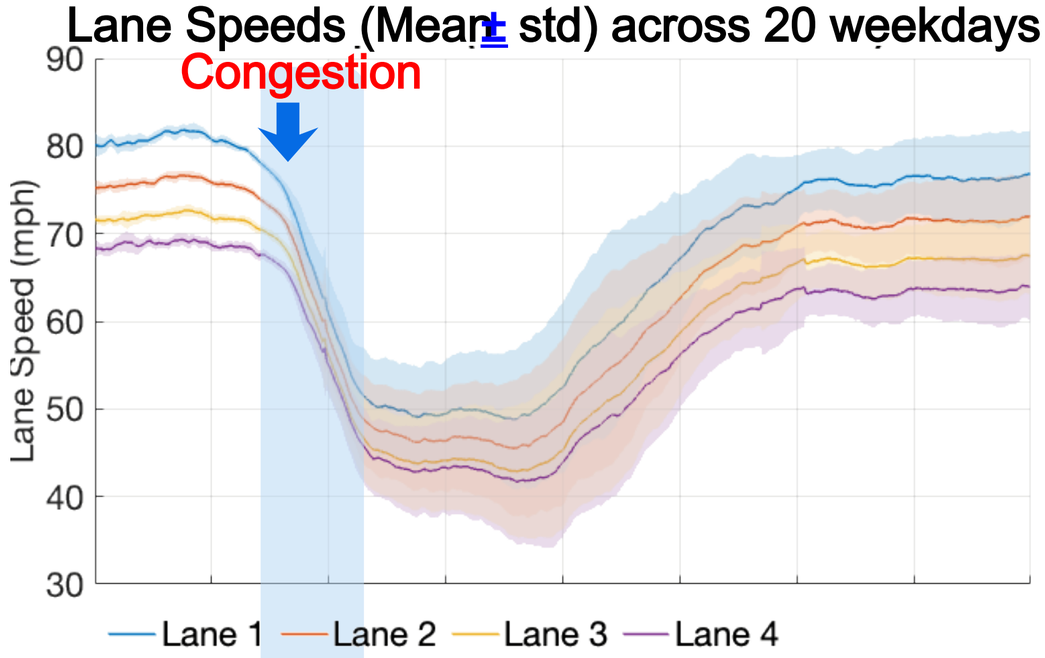
Key question : Does the network pattern change before average traffic speed drops?

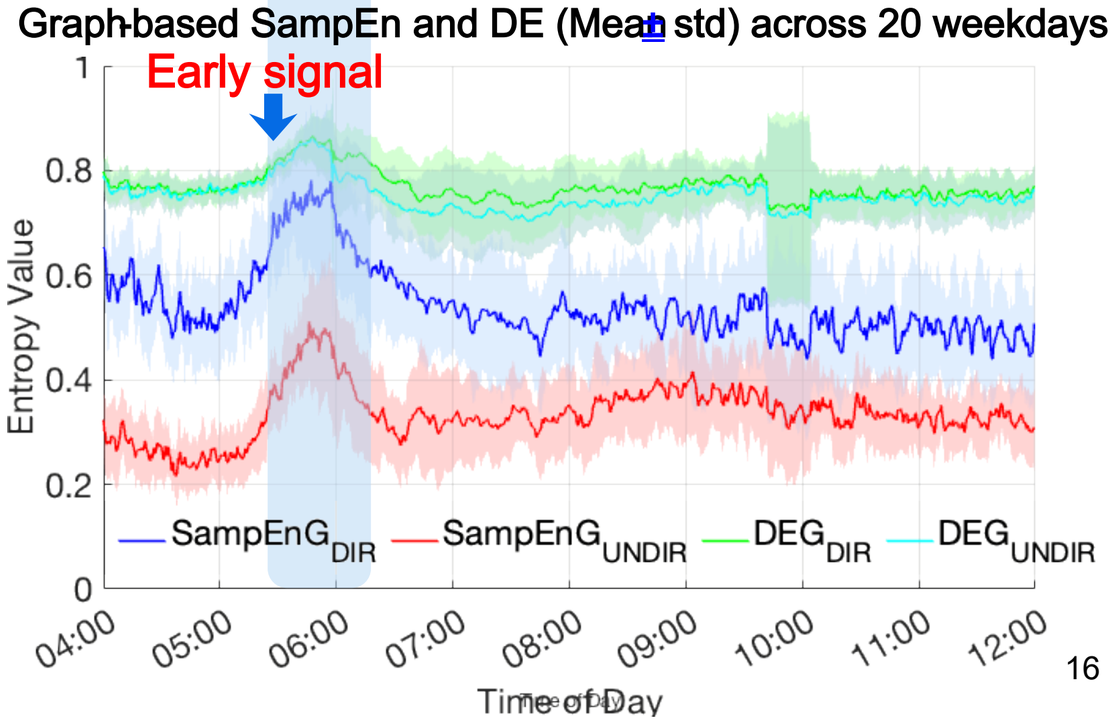
Entropy on graph signals reveals early congestion dynamics

- Graph Dispersion Entropy (DE) (green & cyan)
- Graph Sample Entropy (SampEn)

  (blue & red-on undirected and directed graphs)

Graph-based SampEn & DE increases before traffic speed drops → state transition for congestion





CONCLUSION

Graph - based Sample entropy

- First measure of nonlinear activities for graph signals based on *conditional entropy*.
- Generalises 1D, 2D sample entropy.
- Detects state transition on networked system.

Future work

- Application to other signal types, graph connections.
- Applications to other complex systems such as brain activitiy data.
- Extension to higher-order networks.

THANK YOU!

Presentation by Maggie Lei

Email: M.S.Lei@sms.ed.ac.uk

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