

Locally Stationary Time-Vertex Process Models

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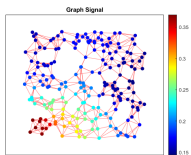
- 1 Introduction
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- 3 Experimental Results
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From Graphs to Time-Vertex Graph Signals

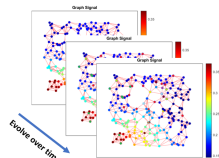
- Many modern datasets arise from **interconnected systems**.
- **Graphs** provide a natural representation for **irregular, non-Euclidean data**.
- **Graph signals** assign data values to nodes. When these values change over time, we obtain **time-vertex** signals.



Interconnected systems
(graph representation)



Graph signal



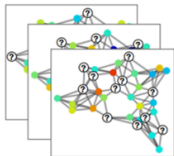
Time-vertex signal

Inference in Time-Vertex Domain

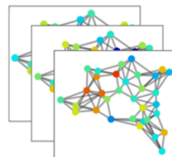
- In many real-world applications, observations are **partial, noisy, or missing**.
- This motivates the need for time-vertex signal models that enable interpolation and reconstruction under incomplete observations.

How can we model and recover time-vertex signals when only a subset of samples is observed?

Partially Observed Signals



Reconstructed Signals



Brief Overview of Time-Vertex Inference Methods

- **Interpolation-based methods**
 - Smoothness priors over time and graph [1], [2]
 - Unrolling methods [3]
- **Graph Neural Networks (GNNs)**
 - Time-vertex GNNs [4], [5]
- **Stochastic process models**
 - Joint time-vertex stationarity [6]

[1] J. H. Giraldo, A. Mahmood, B. Garcia-Garcia, D. Thanou, and T. Bouwmans, "Reconstruction of time-varying graph signals via Sobolev smoothness," *IEEE Transactions on Signal and Information Processing over Networks*, 2022.

[2] A. Javaheri, A. Amini, F. Marvasti, and D. P. Palomar, "Learning spatiotemporal graphical models from incomplete observations," *IEEE Transactions on Signal Processing*, 2024.

[3] M. Nagahama, K. Yamada, Y. Tanaka, S. H. Chan, and Y. C. Eldar, "Graph signal restoration using nested deep algorithm unrolling," *IEEE Transactions on Signal Processing*, 2022.

[4] S. Hadou, C. I. Kanatsoulis, and A. Ribeiro, "Space-time graph neural networks," in *Proc. International Conference on Learning Representations (ICLR)*, 2022.

[5] J. A. Castro-Correa, J. H. Giraldo, M. Badiely, and F. D. Malliaros, "Gegenbauer graph neural networks for time-varying signal reconstruction," *IEEE Transactions on Neural Networks and Learning Systems*, 2024.

[6] A. Loukas and N. Perraudin, "Stationary time-vertex signal processing," *EURASIP Journal on Advances in Signal Processing*, 2019.

Parametric Stochastic Time-Vertex Models

- Stochastic process models: Realistic in most scenarios, interpretable
- Parametric processes: Less complex learning problem, advantageous in data-scarce inference applications
- Extension of classical **MA** and **ARMA** models to time-vertex signals has been used in several works [7], [8]:

$$\mathbf{x}_t = \sum_{p=1}^P \sum_{k=0}^K a_{pk} \mathbf{L}_G^k \mathbf{x}_{t-p} + \sum_{q=0}^Q \sum_{m=0}^M b_{qm} \mathbf{L}_G^m \mathbf{w}_{t-q},$$

- These models typically rely on **global stationarity** assumptions, meaning that the same model parameters apply uniformly across all nodes and time instants [9].

[7] E. Isufi, A. Loukas, N. Perraudin, and G. Leus, "Forecasting time series with VARMA recursions on graphs," *IEEE Trans. on Signal Processing*, 2019.

[8] E. T. Guneyi et al., "Learning graph ARMA processes from time-vertex spectra," *IEEE Transactions on Signal Processing*, 2024.

Limitations of Global Stationarity

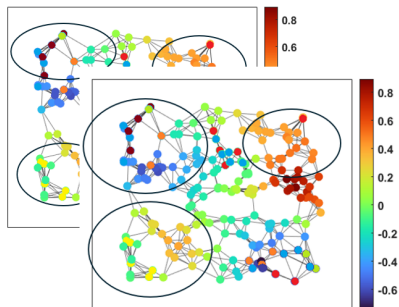
- Global stationarity assumes **homogeneous behavior** across all nodes and time instants.
- Real-world time-vertex signals often exhibit **spatial heterogeneity**, **temporal non-stationarities**, and **localized structural changes**.
- While statistically grounded and interpretable, global models may fail to capture **local variations** in heterogeneous time-vertex data.

Consequence

Degraded reconstruction and interpolation performance in heterogeneous time-vertex settings.

Motivation

This gap motivates the proposed **Locally Stationary Time-Vertex Process (LSTVP)** model.

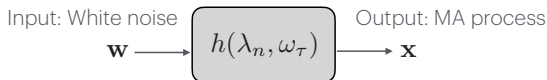


Globally Stationary MA Processes

- Let's first recall global Moving Average (MA) time-vertex process models:

$$\mathbf{x}_t = \sum_{q=0}^{Q-1} \sum_{m=0}^{M-1} b_{qm} \mathbf{L}_G^m \mathbf{w}_{t-q}$$

- $\mathbf{x}_t \in \mathbb{R}^N$: MA process at time t
 - $\mathbf{w}_t \in \mathbb{R}^N$: white noise process at time t
 - $\mathbf{L}_G \in \mathbb{R}^{N \times N}$: graph Laplacian matrix
 - $\{b_{qm}\}$: polynomial graph filter coefficients
- This can be expressed as a filtering operation in the spectral domain:



Joint time-vertex filter in spectral domain:

$$h(\lambda_n, \omega_\tau) = \sum_{q=0}^{Q-1} \sum_{m=0}^{M-1} b_{qm} \lambda_n^m e^{-j\omega_\tau q}$$

Extension to Locally Stationary Models

- Idea: Represent local stationarity as a superposition of K local components
(Builds on previous work [9] which studies local vertex stationarity)

$$\mathbf{x} = \sum_{k=1}^K \underbrace{\mathbf{G}_k \mathbf{U}_J \underbrace{\mathbf{H}_k \underbrace{\mathbf{U}_J^H \mathbf{w}}_{\text{JFT}}}_{\text{Joint filtering}}}_{\text{IJFT}}_{\text{Locality by membership}}$$

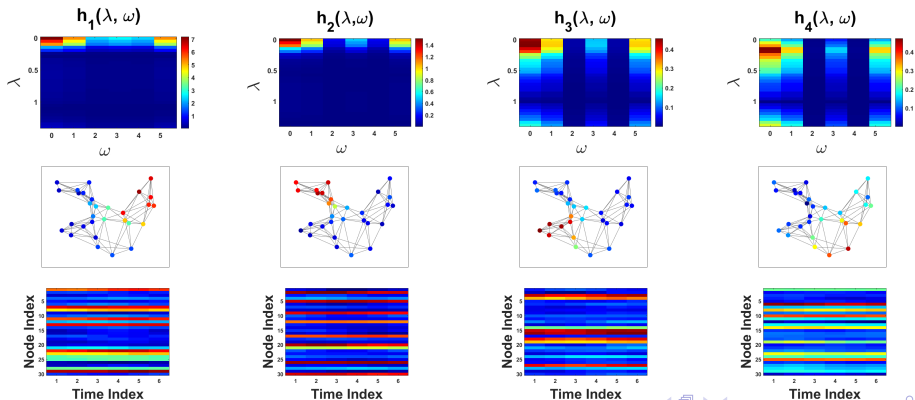
- \mathbf{G}_k : diagonal membership matrix
- \mathbf{U}_J : joint Fourier basis
- \mathbf{H}_k : diagonal matrix representing the time-vertex MA filter
 $h_k(\lambda_n, \omega_\tau) = \sum_{q=0}^{Q-1} \sum_{m=0}^{M-1} b_{qmk} \lambda_n^m e^{-j\omega_\tau q}$.
- \mathbf{w} : white noise process

[9] A. Canbolat, E. Vural, "Locally Stationary Graph Processes", *IEEE Transactions on Signal Processing*, 2024.

Locally Stationary Time-Vertex Processes

- Proposed LSTVP model: $\mathbf{x} = \sum_{k=1}^K \mathbf{G}_k \mathbf{U}_J \mathbf{H}_k \mathbf{U}_J^H \mathbf{w}$
- **Joint spectral filter** \mathbf{H}_k : Captures frequency characteristics $h_k(\lambda_n, \omega_\tau)$ of k -th local component process
- **Membership matrix** \mathbf{G}_k : Activates k -th local component over a certain time window and graph region, enabling **localized** and **overlapping** representations.

Examples of typical joint spectral filters and corresponding membership values:



Learning: Handling Complexity and Non-Convexity

- **Model parameters to learn:** Polynomial filter coefficients $\{b_{qmk}\}$ and membership values \mathbf{G}_k
- To reduce model complexity, we adopt a **low-dimensional** membership representation (following the idea in [10]):

$$\mathbf{G}_k = \text{diag}(\mathbf{D}\mathbf{v}_k), \quad \mathbf{D} \in \mathbb{R}^{NT \times P}, \quad \mathbf{v}_k \in \mathbb{R}^P.$$

- **D:** Tall “dictionary” matrix ($P \ll NT$), with columns representing typical time-vertex localization patterns learnt offline
- Learning $\mathbf{G}_k \equiv$ Learning coefficients \mathbf{v}_k
- Let $\mathbf{v} \in \mathbb{R}^{PK}$ contain all membership coefficients $\{\mathbf{v}_k\}$
Let $\mathbf{b} \in \mathbb{R}^{QMK}$ contain all polynomial filter coefficients $\{b_{qmk}\}$
- To deal with non-convexity, define lifted matrices:

$$\mathbf{V} \triangleq \mathbf{v}\mathbf{v}^T \in \mathbb{R}^{PK \times PK}, \quad \mathbf{B} \triangleq \mathbf{b}\mathbf{b}^T \in \mathbb{R}^{QMK \times QMK}$$

[10] A. Canbolat, E. Vural, “An efficient algorithm for learning locally stationary graph processes,” *EUSIPCO*, 2024.

LSTVP Models: Learning Algorithm

- Optimization problem for learning LSTVP model:

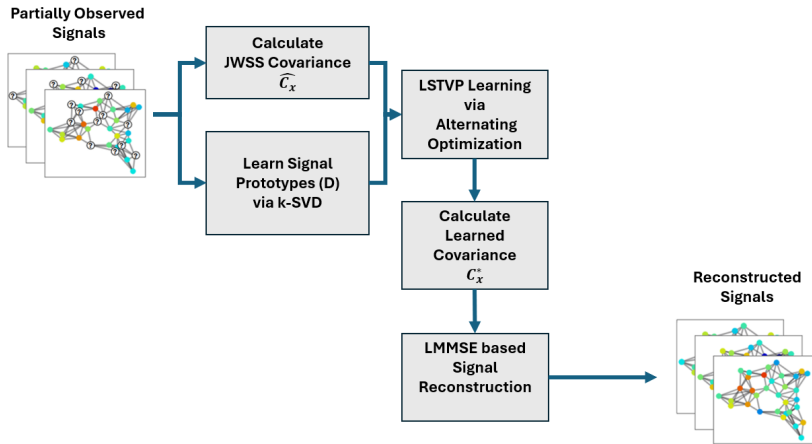
$$\begin{aligned} \min_{\mathbf{V}, \mathbf{B}} \quad & \left\| \mathbf{C}_x(\mathbf{V}, \mathbf{B}) - \widehat{\mathbf{C}}_x \right\|_F^2 + \mu_1 r(\mathbf{V}) + \mu_2 \text{tr}(\mathbf{V}) + \mu_3 \text{tr}(\mathbf{B}) \\ \text{s.t.} \quad & \mathbf{V} \succeq \mathbf{0}, \quad \mathbf{B} \succeq \mathbf{0} \end{aligned}$$

- $\left\| \mathbf{C}_x(\mathbf{V}, \mathbf{B}) - \widehat{\mathbf{C}}_x \right\|_F^2$: Fits model covariance $\mathbf{C}_x(\mathbf{V}, \mathbf{B})$ to empirical covariance $\widehat{\mathbf{C}}_x$
- $r(\mathbf{V}) = \sum_{k=1}^K \mathbf{v}_k^\top \mathbf{D}^\top \mathbf{L}_J \mathbf{D} \mathbf{v}_k$: Penalizes rapid variations of memberships $\mathbf{g}_k = \mathbf{D} \mathbf{v}_k$ over time and graph through joint Laplacian $\mathbf{L}_J \in \mathbb{R}^{NT \times NT}$
- $\text{tr}(\mathbf{V})$ and $\text{tr}(\mathbf{B})$: Promote low-rank structure for matrices $\mathbf{V} = \mathbf{v} \mathbf{v}^\top$, $\mathbf{B} = \mathbf{b} \mathbf{b}^\top$

Solution Method

- Problem is not jointly convex, but individually convex in \mathbf{V} and \mathbf{B} .
- Alternating minimization algorithm:
 - Fix \mathbf{V} , optimize \mathbf{B}
 - Fix \mathbf{B} , optimize \mathbf{V}
- Output: Find LSTVP model parameters $\{\mathbf{v}, \mathbf{b}\}$ via rank-1 decomposition $\mathbf{V} = \mathbf{v} \mathbf{v}^\top$, $\mathbf{B} = \mathbf{b} \mathbf{b}^\top$

Overall Learning Pipeline



An overview of the proposed learning procedure from incomplete observations to parameter estimation.

Experimental Results: Setting

- **Datasets**

- Synthetic time-vertex signals generated via LSTVP model
- Real-world datasets:
 - Molène dataset [11]
 - NOAA dataset [12]

- **Baseline methods**

- **Joint Wide-Sense Stationarity (JWSS)** [6]
- **Global joint MA** [8]
- **Global joint ARMA** [8]

- **Evaluation metrics (signal reconstruction)**

- Normalized Mean Error (NME)
- Mean Absolute Error (MAE)

[6] A. Loukas and N. Perraudin, "Stationary time-vertex signal processing," *EURASIP Journal on Advances in Signal Processing*, 2019.

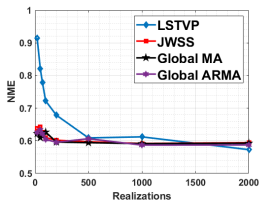
[8] E. T. Guneyi et al., "Learning graph ARMA processes from time-vertex spectra," *IEEE Transactions on Signal Processing*, 2024.

[11] B. Girault, "Stationary graph signals using an isometric graph translation," in *EUSIPCO*, 2015.

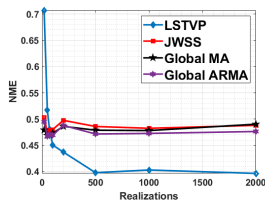
[12] A. Arguez et al., "NOAA's 1981–2010 U.S. climate normals: An overview," *Bulletin of the American Meteorological Society*, 2012.

Performance Analysis: Effect of Realizations

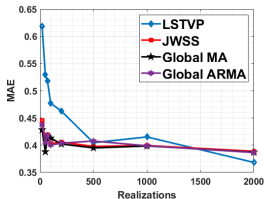
- Evaluation on synthetic data: Learn model from training signals, estimate missing observations (25%) in test signals. ($K = 4$, $N = 30$, $T = 6$)



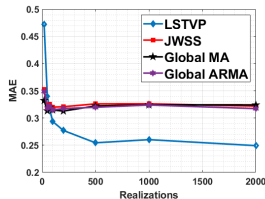
(a) NME vs. Realizations (SNR: 0 dB)



(b) NME vs. Realizations (SNR: 10 dB)



(c) MAE vs. Realizations (SNR: 0 dB)

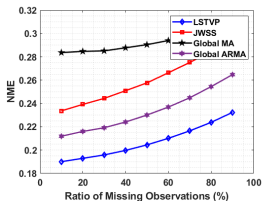


(d) MAE vs. Realizations (SNR: 10 dB)

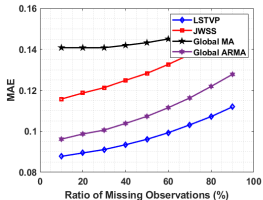
Comparative Experiments

- Evaluation on weather data sets: Learn model from training data, estimate missing observations of test data.

Molène: $N = 37$, $T = 12$, Train/Test=72/660

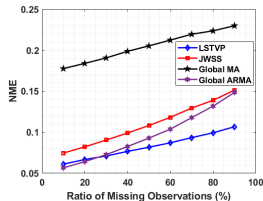


NME - Molène

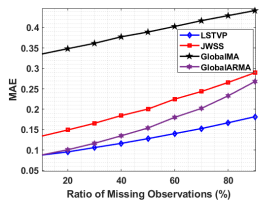


MAE - Molène

NOAA: $N = 65$, $T = 12$, Train/Test=83/870

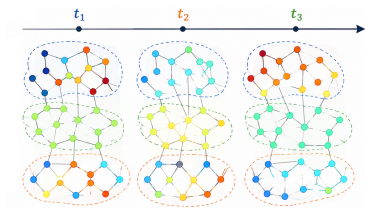


NME - NOAA



MAE - NOAA

Conclusion and Contributions



- We proposed a **Locally Stationary Time-Vertex Process (LSTVP)** model for graph signals evolving over time.
- The model captures **local spatio-temporal variations** while preserving a **parametric and interpretable structure**.
- A learning framework enables reliable interpolation under **noise** and **missing observations**.
- Future work: Extension to settings where the underlying graph structure is **unknown** or **dynamically evolving**.